LIQUID-DROP MODEL FOR EXTREMELY NEUTRON RICH NUCLEI

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Nuclear energy levels are characterized in part by their isospin quantum numbers. Ordinary nuclides are well described by an independent-particle model with ground-state isospins equal to the minimum possible value $T_{min} = abs(A/2 - Z)$. It has been suggested that extremely neutron rich nuclei constitute a second branch of the table of isotopes whose ground states have the maximum possible isospin $T_{max} = A/2$ and that neutral members of the T_{max} branch (i.e., polyneutrons) serve as mediating particles for the new class of nuclear reactions discovered by Fleischmann and Pons. The energetics of the new reactions have been qualitatively described by a liquid-drop model. Recent measurements of the mass spectrum of reaction products produced in the new reactions make possible a refinement of the model, providing an explanation for gaps of instability separating ranges of stability in the mass spectrum.

INTRODUCTION

It is well established experimentally and theoretically that reactions between charged nuclei cannot occur at ordinary temperatures and pressures. A kinetic or potential energy boost on the order of a million electron volts is required to overcome the Coulomb barrier. This level of kinetic energy is not available at the low temperatures of the new class of nuclear reactions discovered by Fleischmann and Pons¹ (cold fusion reactions), nor is sufficient potential energy available from the distortions of crystalline material in which the nuclei may be embedded. These energies fall many orders of magnitude short of what is required.

The situation is different for reactions between a charged nucleus and a neutral particle. For these reactions there is no Coulomb barrier to prevent the two from coming close enough to react. Conventional nuclear reactors exploit this situation with neutrons as the neutral particles. Although neutrons are unstable and decay to hydrogen nuclei in a few minutes, a single stray neutron in a properly designed reactor can initiate a chain reaction that generates vast numbers of neutrons in a very short time. Also, vast quantities of energy can be generated from the associated nuclear reactions.

We know that neutrons cannot be responsible for cold fusion reactions because they are seldom observed—and never in sufficient numbers. Some other neutral particle is required. We note that these reactions are unlike any of the nuclear processes with which we are familiar, and we should not be surprised if this revolutionary discovery turns out to require a revolutionary explanation. To recapitulate, if cold fusion reactions do in reality occur, they cannot occur between charged nuclei and must be mediated by neutral particles other than neutrons. There are two possibilities. The neutral particles may be polyneutrons (groups of neutrons bound together by nuclear forces at densities comparable with those in charged nuclei), or they may be exotic new particles beyond the ken of current physics.

This paper explores the possibility that the required neutral particles are polyneutrons. The binding energies of polyneutrons are quantified in a liquid-drop model. Model parameters are determined from the results observed in selected cold fusion reactions. In this way, many such reactions can be quantitatively understood. More importantly the model points to a number of clear-cut predictions that can be experimentally confirmed or refuted. If refuted, the polyneutron model will have to be discarded, and a neutral particle new to physics must be sought. If on the other hand the polyneutron model can be confirmed, as seems likely based on its ability to explain so much of what we see, it will open up a new branch of nuclear physics.

The possibility that neutrons may be bound into polyneutrons has been considered before. Experimentally, the dineutron ${}^{2}n$ is known not to be bound,² but it fails of binding by only ~0.1 MeV so that a collection of dineutrons bound together by pair-binding of the type elucidated by Bardeen, Cooper, and Schrieffer³ (BCS)

in their theory of superconductivity is likely to be bound. Calculations based on interaction potentials deduced from ordinary nuclei, neglecting the additional binding expected from the BCS correlation, have suggested that polyneutrons smaller than $\sim 100 n$ are not bound. With the BCS interaction included, we can expect binding of smaller aggregates. The present analysis suggests that ${}^{6}n$ may be the smallest bound polyneutron.

Polyneutrons cannot persist in nature because their constituent neutrons are unstable. In successive beta decays, they transmute through the chain ${}^{A}n \rightarrow {}^{A}H \rightarrow {}^{A}He$ and so on until the buildup of Coulomb energy makes further decay energetically unfavorable. Polyneutrons ^{A}n have isospin T = A/2 with three-component $T_3 = -A/2$. Replacement of a neutron by a proton transforms ^{A}n to massive hydrogen ^AH and changes the three-component to $T_3 = -A/2 + 1$, but it leaves total isospin of the ground state unchanged at T = A/2. Total isospin does not decrease because a change would require disruption of the nuclear pair correlation, which in turn would require an increase in energy comparable with the energy gap in the BCS excitation spectrum. All beta decay descendants are members of a single isospin multiplet. Together, polyneutrons and their beta decay descendants constitute a second, extremely neutron rich, branch of the Table of Isotopes.

The energy release in cold fusion reactions shows that large numbers of polyneutrons must participate, on the order of 10^{10} /J of energy production. This raises a serious question as to where they come from. In the picture presented in this paper, they all come from a single initial polyneutron that, like a single neutron in a conventional nuclear reactor, multiplies in a chain reaction to the large number required. Polyneutron reactions are more complex than neutron reactions because polyneutrons can grow in size by picking up neutrons from charged nuclei. As a result, such reactions lead to a range of sizes, up to a thousand neutrons or larger. In interaction with charged nuclei, the large polyneutrons can fission, providing the multiplication step required for a chain reaction.

Even with polyneutron growth and multiplication to account for a chain reaction and the generation of vast numbers of polyneutrons, it still is necessary to account for the first polyneutron. As we have seen, polyneutrons cannot be natural constituents of matter because they are unstable. We must suppose that stable members of the T_{max} branch of the Table of Isotopes exist in nature⁴ and that an initial polyneutron is generated when a neutron interacts with one of them, $n + {}^{A}X \rightarrow {}^{B}n + {}^{A-B+1}X$. The following analysis identifies a range of isotopes ${}^{A}X$ that are capable of generating polyneutrons in exothermic reactions of this type. Although this process for generating an initial polyneutron requires that stable members of the T_{max} branch be present in nature, there need not be many of them. A very low concentration relative to ordinary isotopes would suffice. The initial polyneutron may be generated in the electrolyte, for example, from T_{max} lithium in a lithium-salt electrolyte or from T_{max} carbon in a carbonate electrolyte. It may be generated in a reactor component, for example, from T_{max} carbon in a rubber O-ring. In contrast, it may be generated outside the reactor, for example, from nitrogen in the air, whence it can diffuse into the reactor.

The nuclear chain reaction suggested here requires polyneutron growth in the electrolyte followed by polyneutron fission in the cathode, followed by growth again in the electrolyte and fission again in the cathode, and so on for many cycles. The ability to support this chain reaction can depend on many factors, including the topology of the electrolyte-cathode interface and the location and severity of the stirring caused by bubbling at the cathode surface. These conditions are very complex, but one can begin to see a way of understanding the erratic and unpredictable results of many experiments where the surface and interface conditions are not well characterized or controlled and where they may change during the course of electrolysis.

Difficult as it may be to accept the idea of a second, and until the present, unnoticed, branch of the Table of Isotopes, of which the neutral members are the polyneutrons here described, an explanation at least as revolutionary as this is required if cold fusion reactions are to be understood and united with the existing body of nuclear knowledge.

POLYNEUTRON REACTIONS

The analogy between neutron matter and liquid helium has suggested that although neutron pairs and helium atom pairs are not bound, a sufficiently large number of neutrons will form a stable aggregate just as a sufficiently large number of helium atoms will form a liquid drop.² The dineutron fails of binding by $\sim \frac{1}{10}$ MeV. If the interaction of a small cluster of dineutrons were to reduce their mutual energy by several tenths or more of a mega-electron-volt per neutron, they would form a bound droplet of nuclear matter. The BCS correlated-pair interaction favors such binding, which is assumed in the following analysis.

The nuclear processes involved in cold fusion reactions must include a set of exothermic reactions that accomplish the creation of an initial polyneutron, that support polyneutron growth and multiplication, and that support transmutation of ordinary independent-particle isotopes. The following illustrative reactions are capable in principle of meeting these requirements.

An initial polyneutron ${}^{B}n$ can be created in a reaction such as

$${}^{1}n + {}^{A}X \to {}^{B}n + {}^{A-B+1}X , \qquad (1)$$

where ${}^{A}X$ and ${}^{A-B+1}X$ are T_{max} isotopes with ${}^{A}X$ assumed present in nature and where (in experiments to date) the

initiating neutron is a stray from the environment. [The following analysis shows that reaction (1) is exothermic if ^{*A*}X is any of the following stable T_{max} isotopes: lithium with mass number A = 35, 37, and 39; beryllium with A = 53, 55, 57, 59, 61, and 63 through 76; boron with A = 106 through 130; carbon with A = 187 through 208; nitrogen with A = 304 through 313; and possibly hydrogen with A = 6.] Polyneutron growth can be supported in experimental systems containing lithium by the reactions

 $^{A}n + ^{7}\text{Li} \rightarrow ^{A+2}n + ^{5}\text{Li}$

and

$$^{E+1}n + {}^{6}\text{Li} \rightarrow {}^{E+2}n + {}^{5}\text{Li} \quad . \tag{2b}$$

(2a)

where *E* is an even integer. The ⁵Li promptly decays to ${}^{4}\text{He} + {}^{1}\text{H}$, providing a mechanism for the generation of helium in cold fusion reactions as observed by Miles et al.⁵ For appropriate ranges of *A* and *B*, polyneutrons can multiply in reactions such as

$$^{A+B+6}n + {}^{102}\text{Pd} \rightarrow {}^{A}n + {}^{B}n + {}^{108}\text{Pd}$$
 (3a)

and

$$^{A+B+6}n + {}^{58}Ni \rightarrow {}^{A}n + {}^{B}n + {}^{64}Ni$$
 . (3b)

Note that a chain reaction is supported by reactions (2a) and (2b) together with either of reactions (3a) or (3b). Note also that polyneutrons can grow extremely large by repetition of reactions (2a) and (2b). Isotopes of independent-particle nuclides can be transmuted in reactions such as

$${}^{A+4}n + {}^{105}\text{Pd} \to {}^{A}n + {}^{109}\text{Pd} \\ {}^{109}\text{Pd} \to {}^{109}\text{Ag} (13.7 \text{ h})$$
 (4a)

$${}^{A+4}n + {}^{61}\text{Ni} \to {}^{A}n + {}^{65}\text{Ni} \\ {}^{65}\text{Ni} \to {}^{65}\text{Cu} (2.5 \text{ h}) \right\} .$$
 (4b)

The liquid-drop model provides a quantitative treatment that shows all of these reactions to be exothermic.

Because of the very weak binding of an unpaired neutron to the BCS correlated-pair state, we expect that for even *E* the mass excess of the polyneutron ^{E+1}n will be larger than that of ^{E}n by an amount approaching the mass excess of a free neutron. The reaction $^{E}n + {}^{6}\text{Li} \rightarrow {}^{E+1}n + {}^{4}\text{He} + {}^{1}\text{H}$ then is endothermic, and the generation of odd-*A* polyneutrons by interaction with lithium is not energetically accessible. As a consequence, we expect that polyneutrons growing on lithium will have even *A* as in reactions (2a) and (2b).

In contrast, odd-A polyneutrons and odd-A hydrogen in the T_{max} configuration can be generated when even-A polyneutrons interact with ordinary hydrogen (here E1 and E2 are both even integers):

$$E_{1+E_{2}} = 1 \text{ II} = \int_{-\infty}^{E_{1}} n + E_{2+1} \text{ II}$$
 (5a)

$$\sum_{n \to -n} e^{-n} + e^{-n} \rightarrow \sum_{n \to -n} e^{-n} e^{-n} + e^{-n} e^{-n} + e^{-n} e^{-$$

With mass excess values from the liquid-drop model developed below, these reactions are exothermic for appropriate ranges of E1 and E2 but less exothermic than reaction (2). Because they are less exothermic, we expect them to be slower than reaction (2), with the consequence that polyneutrons on balance are able to grow very large in the electrolyte.

Whenever an odd-*A* polyneutron is produced in reaction (5b), it shortly thereafter loses its odd neutron in one of many reactions then energetically available. In an electrolyte containing heavy water, for example, we can have

$$^{E+1}n + {}^{2}\mathrm{H} \rightarrow {}^{E}n + {}^{3}\mathrm{H} , \qquad (6)$$

accounting for the generation of tritium observed in such systems.⁶

From reactions (5a) and (5b) we expect approximately equal numbers of odd-*A* and even-*A* isotopes of hydrogen. Because of the Coulomb barrier, they cannot participate in reactions with other charged nuclides, but as will be shown, they undergo beta decay and fission until only stable daughter T_{max} nuclides remain.

REVIEW OF MASS SPECTROMETRY EVIDENCE

Beginning with Fleischmann and Pons,¹ the earliest experiments involving cold fusion reactions were concerned with the generation of energy in amounts too great to be attributable to chemical reactions or experimental uncertainties. More recent experiments have been concerned with the properties and identities of nuclear reaction products. Neutron activation analysis and other techniques of analysis that enable identification of ordinary nuclides are not at present capable of identifying T_{max} nuclides because the excitation spectra of T_{max} nuclides are unknown. Direct detection by mass spectrometry is required. The secondary ion mass spectrometry experiments by Miley et al.⁷ and of Mizuno et al.⁸ are of particular utility. These sets of experiments are fully capable of detecting T_{max} isotopes, and they corroborate each other in finding significant evidence for a wide range of nuclear reaction products.

Reactor volume in the Miley experiments is $\sim 1 \text{ cm}^3$. The reactor contains about a thousand small plastic or glass beads coated with one or more thin layers of nickel or palladium or both. An electrolyte of Li₂SO₄ in light water is recirculated through the reactor, and hydrogen is deposited on the metallic surfaces of the beads by concurrent electrolysis. The electrodes are generally made of titanium. Nuclear energy generation amounts to a few watts. No gamma-ray emission is observed, and no neutrons are observed. After typically several weeks of reaction, secondary ion mass spectrometry shows that reaction products with a wide range of masses have been generated.

Summary mass spectra for six experimental runs conducted by Miley et al.⁹ are reproduced in Fig. 1. The vertical scale in the figure is logarithmic. Practically all atoms in the mass spectra are concentrated in discrete mass regions, as summarized in Table I.

The results shown in Fig. 1 defy explanation in terms of cold nuclear reactions involving solely the familiar independent-particle nuclides. Ordinary nuclei cannot interact without neutron mediation, cannot generate a wide spectrum of reaction products without emission of gamma rays, and cannot generate reaction products more massive than the reactants. The way is open for explanation in terms of T_{max} nuclides.

In the analytic treatment to follow, the boundaries of the regions containing the vast majority of isotopes are interpreted as the limits of stability of the isotopes in each region. The finite lifetimes of unstable isotopes can blur the boundaries of stability. The nearer an unstable isotope lies to the boundary, the longer is its half-life and the greater the likelihood of its survival for detection in mass spectrometry. This introduces uncertainty when the limits of stability are quantified because the residual population of unstable isotopes near the bounds of stability

TABLE I

Ranges of Stability for Nuclear Reaction Products in Fig. 1*

Nuclear Charge Z	Stability Range A
1 2 3 4 5 6	$ \begin{cases} 6 to 34 \\ 52 to 82 \\ 106 to 130 \\ 190 to 208 \end{cases} $

*Each range contains mass numbers for stable T_{max} isotopes of a single element. A nuclear charge has been assigned to each range as described in the text.

will depend on the length of time between run termination and mass determination. The upper bounds of the regions of stability are determined by beta decay and are expected to be fairly sharply defined. The lower bounds may be less sharply defined because they are determined by fission, which is expected to have a longer lifetime



Fig. 1. Shown is a summary of production rates of long-lived isotopes in cold fusion reactions, determined by mass spectrometry following the end of each run.⁹ It is proposed that most of these isotopes belong to a neutron-rich T_{max} branch of the Table of Isotopes. Mass number ranges with high production rates are interpreted as ranges of T_{max} stability. Interposed ranges with low production rates are interpreted as regions of instability where T_{max} isotopes have short lives and disappear before they can be detected.

because of the Coulomb barrier. The presence of ordinary independent-particle isotopes in the mass spectrum may also affect the definition of the T_{max} regions, as may experimental background uncertainties that follow from the small sample size.

LIQUID-DROP MODEL

Liquid-drop models have proven useful for parameterizing the masses of ordinary nuclei (and of atoms when the masses of the electrons are included). A typical model¹⁰ for atomic mass excess as it depends on mass number A and nuclear charge Z for ordinary atoms is

$$\begin{split} \Delta(A,Z) &= (A-Z)\Delta(n) + Z\Delta(^{1}\mathrm{H}) - a_{v}A + a_{s}A^{2/3} \\ &+ a_{t}(A/2-Z)^{2}/A + a_{c}Z(Z-1)A^{-1/3} \\ &+ a_{o}(A,Z) \ , \end{split}$$

where

 $(A - Z)\Delta(n) =$ number of neutrons (A - Z)times the neutron mass excess

- $Z\Delta(^{1}H) =$ number of protons times the hydrogen atom mass excess
 - $a_v A$ = nuclear binding energy term proportional to the number of nucleons
- $a_s A^{2/3}$ = surface energy term proportional to the surface area of the nucleus
- $a_t(A/2 Z)^2/A =$ energy that increases in proportion to $(N Z)^2/A$, reflecting a quadratic dependence of energy on isospin

$$a_c Z(Z-1)A^{-1/3}$$
 = Coulomb energy

 $a_o(A,Z)$ = energy term that depends on whether *N* and *Z* are even or odd.

The model must be modified for T_{max} isotopes. The $a_t(A/2 - Z)^2/A$ term must be dropped because energy depends linearly on isospin in the neighborhood of T = A/2. Nothing need be substituted for it because states with isospin A/2 - 1 are currently inaccessible.

The $a_o(A, Z)$ term now depends only on whether A is even or odd. For odd A this term is simply a_o , and for even A it is zero.

The Coulomb term $a_c Z(Z-1)A^{-1/3}$ must be modified to take account of proton pair correlations in the BCS state. For a nuclide with A nucleons and Z protons, the number of distinct proton pairings is Z(Z-1)/2, and the number of distinct nucleon pairings is A(A-1)/2. For even A, every fully paired configuration contains A/2pairs. The mean number of proton pairs in the nucleus then is

$$\frac{Z(Z-1)/2}{A(A-1)/2} \left(\frac{A}{2}\right) = \frac{Z(Z-1)}{2(A-1)} \quad . \tag{8}$$

Because of the attractive pair interactions in the BCS state, there is an elevated probability of finding two nucleons close to each other. The $Z(Z-1)A^{-1/3}$ term for Coulomb energy does not recognize this elevated probability, which adds an increment of Coulomb energy proportional to the number of proton pairs. When the proton-pair term is included, the Coulomb energy term becomes $Z(Z-1) \times$ $[c_1A^{-1/3} + c_2(A-1)^{-1}]$. Although derived for even *A*, this expression is also used for odd *A*.

The nuclear binding energy can be represented as the sum of a linear part proportional to the number of nucleons A plus a nonlinear part that is a more complex function of A. For large nuclei, we expect that the two-parameter formula $-a_vA + a_sA^{2/3}$, as fit to mass values for large nuclei, will provide a good representation of nuclear binding energy for the BCS state. However, a correction to the nonlinear part is required for the smallest nuclei to allow for a weakening of nuclear binding energy relative to that provided by the two-parameter formula. The correction employed in this analysis is substitution of $(A + c_3)^{2/3}$ for $A^{2/3}$ and addition of a term $\delta(A)A$ to provide additional weakening for very small A.

With the foregoing modifications, the liquid-drop model for the mass excess of T_{max} atoms with even A is

$$\Delta(A,Z) = (A-Z)\Delta(n) + Z\Delta(^{1}\mathrm{H})$$

- $[a_{v} - \delta(A)]A + a_{s}(A + c_{3})^{2/3}$
+ $Z(Z-1)[c_{1}A^{-1/3} + c_{2}(A-1)^{-1}]$, (9a)

and for atoms with odd (A + 1), the mass excess is

$$\Delta(A+1,Z) = \Delta(A,Z) + a_o \quad . \tag{9b}$$

The model contains five parameters $(a_v, a_s, c_1, c_2, and c_3)$ and two correction terms $\delta(A)$ and a_o —all of which must be determined by appeal to the experimental data.

DETERMINATION OF PARAMETER VALUES

The Coulomb energy parameters c_1 and c_2 can be determined from the upper mass bounds of the atomic stability regions indicated in Table I. The most massive atom in each stability region is taken to be marginally stable against beta decay, and the atom with one more neutron is taken to be marginally unstable. For the stability region characterized by nuclear charge Z and for A equal to the mass number of the most massive atom in that region, marginal beta stability requires $\Delta(A, Z) \approx \Delta(A, Z + 1)$. Substituting from the liquid-drop formula, one can reduce this expression to

$$c_1 A^{-1/3} + c_2 (A - 1)^{-1} \approx 0.3912/Z \,\text{MeV}$$
 . (10)

Each (A,Z) combination for a stable atom at the high-mass boundary of a region of stability in Table I leads to an equation in the unknown parameters c_1 and c_2 . The four combinations are reproduced in column 2 of Table II. The four resulting equations are approximately satisfied by $c_1 = 0.357$ and $c_2 = 1.023$. Using these values, the limits of stability given by the model are as shown in column 3 of Table II. The quality of the fit supports the assignments of nuclear charge in Tables I and II. Analysis shows that with any other assignments, the quality of the fit is substantially degraded.

The surface energy parameters a_s and c_3 can be determined from the lower bounds for the stability regions in Table I. The least massive atom in each stability region is taken to be marginally stable against decay by fission, and the atom with one fewer neutron is taken to be marginally unstable. For the stability region characterized by nuclear charge Z and for A equal to the mass number of the least massive atom in that region, marginal stability against fission requires the following:

For every *B*, *Y* combination,

 $\Delta(A,Z) < \Delta(A - B, Z - Y) + \Delta(B,Y)$ and for at least one *B*, *Y* combination, $\Delta(A - 1, Z) > \Delta(A - 1 - B, Z - Y) + \Delta(B,Y)$ (11)

Each (A, Z) combination for a stable atom at the lower mass boundary of a region of stability in Table I leads to an independent criterion for the unknown parameters a_s

TABLE II

Nuclear Charge Dependence of the Upper Limit of Beta Decay Stability for T_{max} Atoms*

	Nucleon Number A at Upper Limit of Beta Stability	
Nuclear Charge Z	Experiment	Model
1		6
2		18
3	34	40
4	82	76
5	130	130
6	208	208
7		313
8		449

*Column 1 gives the nuclear charge. Column 2 gives the corresponding experimental upper limit of stability (from Table I). Column 3 gives the upper limit of stability provided by the liquid-drop model [from Eq. (10)].

TABLE III

Nuclear Charge Dependence of the Lower Limit of Stability		
of T_{max} Atoms*		

Nuclear	Nucleon Number A at Lower Limit of Stability		Limiting
Charge Z	Experiment	Model	Decay Mode
1 2 3 4 5 6 7 8	52 106 190	6 7 19 52 106 187 304 462	${}^{3}\text{H} + n + n$ Electron capture Electron capture Fission Fission Fission Fission Fission

*Column 1 gives the nuclear charge. Column 2 gives the corresponding experimental lower limit of stability (from Table I). Column 3 gives the lower limit of stability provided by the liquid-drop model [from Eq. (11)]. Column 4 gives the decay mode of the isotope with nucleon number just below the limit of stability.

and c_3 . The three (A, Z) combinations are reproduced in column 2 of Table III. The three resulting criteria embodied in Eq. (11) are approximately satisfied by $a_s = 0.079$ and $c_3 = 25$. Using these values, the limits of stability as given by the liquid-drop model are shown in column 3 of Table III.

Still to be determined are the coefficient a_v , the correction factor $\delta(A)$, and the odd-A correction term a_o . In determining these, we require first that reactions (1), (2a), (2b), (3a), (3b), (4a), (4b), (5a), and (5b), all related to the polyneutron chain reaction and its side reactions, be exothermic as claimed. Then, because the isotope ¹¹Li undergoes beta decay with a half-life of 8.5 ms—a lifetime so long that it rules out any possibility of an exothermic strong decay—we require that the unobserved reactions

$$\begin{cases} {}^{11}\text{Li} \rightarrow {}^{7}\text{Li} + {}^{4}n , \\ {}^{11}\text{Li} \rightarrow {}^{6}\text{Li} + {}^{5}n , \\ {}^{11}\text{Li} \rightarrow {}^{4}\text{He} + {}^{1}\text{H} + {}^{6}n , \\ {}^{11}\text{Li} \rightarrow {}^{3}\text{He} + {}^{1}\text{H} + {}^{7}n , \end{cases}$$

$$(12)$$

and

 $^{11}\text{Li} \rightarrow {}^{1}\text{H} + {}^{1}\text{H} + {}^{1}\text{H} + {}^{8}n$

be endothermic. Also, because the neutron-rich isotopes ${}^{A}X$ at the high-mass end of the ordinary branch of the Table of Isotopes are observed to decay only by beta

decay, alpha decay, or spontaneous fission, we require that all polyneutron-producing reactions

$$^{A}X \to {}^{A-B}X + {}^{B}n \tag{13}$$

be endothermic. Finally, we require that $\delta(A)$ be negligible for $A \ge 22$ where the variation of $\Delta(A,Z)$ is already accounted for by the parameters a_s , c_1 , c_2 , and c_3 with a constant (but yet to be determined) volumetric coefficient a_v . By trial and error, it was found that all these requirements can be satisfied by the assignments $a_v = 6.5$, $\delta(A) = 0.001(A - 24)^3$ for A < 24, $\delta(A) = 0$ for $A \ge 24$, and $a_o = 7$.

The liquid-drop expression for the mass excesses of T_{max} atoms is now as complete as current experimental data allow. For even *A*,

$$\delta(A) = \begin{cases} 0.001(24 - A)^3 & \text{if } A < 24\\ 0 & \text{if } A \ge 24 \end{cases}$$

and

$$\Delta(A,Z) = 8.071(A - Z) + 7.289Z$$

- [6.5 - $\delta(A$)]A
+ 0.079(A + 25)^{2/3}
+ Z(Z - 1)[0.357A^{-1/3} + 1.023(A - 1)⁻¹]

(14a)

and for odd (A + 1),

$$\Delta(A + 1, Z) = \Delta(A, Z) + 7$$
. (14b)

With this expression for $\Delta(A, Z)$, it can be confirmed that reactions (1), (2a), (2b), (3a), (3b), (4a), (4b), (5a), and (5b) are all exothermic as required. Reaction (1) is exothermic for any stable T_{max} isotope in Table IV with odd $A \ge 35$ or even $A \ge 64$, ranging from ³⁵Li to ³¹³N. Reactions (2a) and (2b) are exothermic for all Aand E, reaction (3a) for $A + B + 6 \ge 42$, reaction (3b) for $A + B + 6 \ge 40$, reaction (4a) for $A + 4 \ge 22$, reaction (4b) for $A + 4 \ge 20$, and reactions (5a) and (5b) for $E1 + E2 \ge 46$.

REACTION CROSS SECTIONS

Very little experimental data are available for evaluating or placing limits on the cross-section areas for polyneutron reactions. The physical size of cold fusion reactors (1 cm³ is adequate) suggests a substantially greater cross section for polyneutrons than for neutrons in interactions with ordinary matter. This is physically plausible because polyneutrons can initiate reactions having a single charged product for which there is no Coulomb barrier, whereas neutrons cannot initiate such reactions. We also can expect a very large cross section for reaction (1), which generates an initial polyneutron, as this reaction

TABLE IV

Nuclear Charge Dependence of the Range of Stability of T_{max} Atoms, from Tables II and III*

Nuclear	Region of Stability	
Charge Z	Experiment	Model
1 2 3 4 5 6 7 8	$ \begin{cases} 6 to 34 \\ 52 to 82 \\ 106 to 130 \\ 190 to 208 \end{cases} $	6 7 to 18 19 to 40 52 to 76 106 to 130 187 to 208 304 to 313

*Gaps of instability separate regions of stability for atoms with a nuclear charge $Z \ge 3$. A narrow region of stability may exist in the neighborhood of $A \approx 308$.

also has a single charged product. The time duration required for initiation depends on the flux of neutrons and the concentration of T_{max} isotopes in addition to the reaction cross section—all of which are unknown. For experiments that rely on stray neutrons, the neutron flux is small and highly variable, depending on the structural components of the laboratory and on the composition and configuration of the experimental setup. The concentration of T_{max} isotopes is certainly small in an initial run. It must be assumed that these are adequately compensated for by the large cross section for reaction (1).

Polyneutron reaction cross sections are expected to be reduced when both products are charged or when there are more than two products. Consider the following three exothermic reactions, listed here in order of their estimated relative cross sections:

$$^{A}n + ^{7}\mathrm{Li} \rightarrow ^{A+2}n + ^{5}\mathrm{Li}$$
 , (15a)

$$^{A}n + ^{7}\text{Li} \rightarrow ^{A+2}\text{H} + ^{5}\text{He}$$
, (15b)

and

$$^{A}n + ^{7}\mathrm{Li} \rightarrow ^{B}n + ^{A-B+2}n + ^{5}\mathrm{Li}$$
 . (15c)

The first of these is reaction (2a), previously considered the primary growth reaction for polyneutrons in interaction with lithium. Competing reaction (15b) with two charged products terminates polyneutron growth by converting neutral ${}^{A}n$ to charged ${}^{A+2}H$, which can no longer participate in cold nuclear reactions. For polyneutrons to grow as large as $\sim 10^{3}$ neutrons, as required for the physical interpretation of the data in Fig. 1, the cross section for reaction (15b) must be $\sim 10^{-3}$ times that of reaction (15a). Reaction (15c) must have a still smaller cross section because otherwise polyneutrons in metallic lithium would multiply faster than they were converted to massive hydrogen, leading to a runaway chain reaction. These relative cross sections are required on phenomenological grounds. Although they seem plausible, they have yet to be verified by direct measurement or by calculation from first principles.

Two other exothermic reactions listed in order of their relative cross sections are

$$^{A+B+6}n + {}^{102}\mathrm{Pd} \rightarrow {}^{A+B+4}n + {}^{104}\mathrm{Pd}$$
 (16a)

and

$$^{A+B+6}n + {}^{102}\text{Pd} \rightarrow {}^{A}n + {}^{B}n + {}^{108}\text{Pd}$$
 . (16b)

The second of these is reaction (3a), previously considered as the primary multiplication reaction for polyneutrons in interaction with palladium. Competing reaction (16a) with only two products is expected to have a larger cross section, with the consequence that polyneutron multiplication is anticipated to be relatively rare compared with shrinkage. Although illustrated here for palladium, the probability of shrinkage is expected to be greater than that of multiplication for polyneutrons in interaction with any material that supports multiplication. Also, for some materials there is no limit to the shrinkage that can take place. A polyneutron confined to nickel will shrink until finally only one or two neutrons remain and the polyneutron is lost to further reaction. This presents a challenge to reactor design, where excessive shrinkage must be avoided if a chain reaction is to be sustained.

The following picture now emerges for the chain reaction in the Li-Pd system. Polyneutrons in the electrolyte grow to large size at the expense of lithium, and then they diffuse into the cathode where they shrink and occasionally multiply at the expense of palladium. The number of polyneutrons is increased. They then diffuse back into the electrolyte where growth takes over to continue the reaction. A self-sustaining chain reaction can be maintained as long as the rate of polyneutron multiplication exceeds the rate of loss from competing reactions.

DISCUSSION

The mass excess expression derived for T_{max} nuclear matter is not based on fundamental principles but was tailored to fit a selection of experimental data. It is possible that its success is accidental and that it carries no predictive power beyond an ability to recover the same data to which it was fit. The physical plausibility and predictive capability of the T_{max} mass excess expression remain to be demonstrated.

Support for physical plausibility can be obtained by comparing the liquid-drop models for T_{max} nuclear matter and ordinary nuclear matter. The volume, Coulomb, and surface terms are compared in Table V. The volume

TABLE V

Comparison of Liquid-Drop-Model Parameters*

	T_{max}	Independent Particle
Volume term Coulomb term Surface term	$-6.5A \\ 0.357Z(Z-1)A^{-1/3} \\ 0.079(A+25)^{2/3}$	$-15.67A \\ 0.736Z(Z-1)A^{-1/3} \\ 17.23A^{2/3}$

*The T_{max} terms with their associated parameters are taken from Eq. (14), and for ordinary nuclear matter the independentparticle terms and parameters are taken from Ref. 10.

term shows that the binding energy per nucleon is only $\sim 40\%$ as great for T_{max} nuclear matter as for ordinary nuclear matter. The coefficient of the Coulomb term is about half as great, indicating that the density of T_{max} matter is only about an eighth that of ordinary nuclear matter. Also, the surface term indicates that the surface energy should be of T_{max} matter is only $\sim 0.5\%$ as great as that of ordinary matter.

Overall, the interpretation of the cold fusion reaction and of the liquid-drop model that supports it appears promising in light of the physical picture of T_{max} matter to which we have been led and considering the ability of the model to illuminate many hitherto inexplicable results of cold fusion experiments, including the observation of gaps of instability separating regions of stability in the mass spectrum of reaction products. These regions of stability are shown graphically in Fig. 2, where they are compared with the stability limits of ordinary isotopes. Nucleon number and nuclear charge coordinates (A, Z) are highlighted in black for ordinary isotopes that are stable against all decay modes (strong, electromagnetic, beta, and fission). Stable T_{max} isotopes are highlighted with cross-hatching, and T_{max} isotopes that are not stable against fission are left unhighlighted. The regions of cross-hatched shading show clearly the bounds of stability that are expected for T_{max} reaction products and that are exhibited in Fig. 1.

In Fig. 2 all T_{max} isotopes whose most accessible decay path involves multiple beta decay are considered to be stable. This is consistent with the practice for ordinary isotopes and with the observation that double beta decay rates are negligible on a geologic timescale. Massive hydrogen ⁶H is a special case for which the decay ⁶H \rightarrow ⁴He + 2*n* is characterized by $\Delta T_3 = \frac{1}{2}$, which is allowed for beta decay, but also by $\Delta T = 2$, which is not allowed.¹¹ As a result the decay rate for ⁶H is expected to be intermediate between single and double beta decay. It thus is possible that ⁶H may be present in the environment and that an initial polyneutron may be produced by the reaction ¹*n* + ⁶H \rightarrow ¹H + ⁶*n*.

Overall, the model points to new research directions supported by quantitative predictions, including the chemical natures of the products found in cold fusion reactions,



Fig. 2. Shown are valleys of stability for ordinary isotopes and for T_{max} isotopes as provided by the liquid-drop model. Each isotope is characterized by its nucleon number *A* and its nuclear charge *Z*. Each (*A*,*Z*) combination defines a rectangular box. Boxes with (*A*,*Z*) combinations corresponding to stable ordinary isotopes are shaded in black. Boxes corresponding to stable T_{max} isotopes are cross-hatched. Boxes corresponding to T_{max} isotopes that are stable against strong, electromagnetic, and weak decay but not against fission are unshaded.

the possible existence of more massive products near A = 308, the existence in nature of low concentrations of T_{max} isotopes, and the isotopic mass shifts and transmutations to be expected in ordinary nuclides when they participate in cold fusion reactions.

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